Kuwait University Math. 101 November 10, 2007
Dept. of Math. & Comp. Sc. First Midterm Exam Duration 90 Minutes

Calculators and mobile phones are not allowed
Answer the following questions

1. (6 Points) Find the following limits, if they exist

$$(a) \quad \lim_{x \to 1} \frac{\sqrt{5-x}-2}{x-1}$$

$$(b) \quad \lim_{x \to 0} \frac{x \tan 2x}{3x^2 + 2\sin^2 x}$$

2. (2 Points) If $f'(2) = -8$, then find $\lim_{x \to 2} \dfrac{f(x) - f(2)}{x^3 - 8}$.

3. (4 Points) Find the vertical and horizontal asymptotes, if any, for the graph of f, where

$$f(x) = \frac{(x+1)\sqrt{x^2+2}}{x^2 - x}.$$

4. (4 Points) Classify the discontiuities of

$$f(x) = \frac{x^2 - 2x}{|x|\,(x^2 - x - 2)}$$

as removable, jump or infinite.

5. (a) (3 Points) Let $f(x) = x^3 + x^2 - x$.

Use the Intermediate Value Theorem to show that there is a point on the graph of f at which the tangent line is horizontal.

(b) (3 Points) Let $f(x) = x^{\frac{4}{3}} + 4x^{\frac{1}{3}} - 2$. Find the point on the graph of f at which the tangent line is vertical.

6. (3 Points) Find y', if $y = \left[\tan\left(\dfrac{x}{2}\right) + \cos 2x\right]^3$.

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1. a $\lim\limits_{x\to 1}\dfrac{\sqrt{5-x}-2}{x-1}=\lim\limits_{x\to 1}\dfrac{1-x}{(x-1)\left(\sqrt{5-x}+2\right)}=\boxed{-\dfrac{1}{4}}$

1. b $\lim\limits_{x\to 0}\dfrac{x\tan 2x}{3x^2+2\sin^2 x}=\lim\limits_{x\to 0}\dfrac{2\left(\tan 2x\right)/(2x)}{3+2\left(\sin x/x\right)^2}=\boxed{\dfrac{2}{5}}$

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2. $\lim\limits_{x\to 2}\dfrac{f(x)-f(2)}{x^3-8}=\lim\limits_{x\to 2}\dfrac{f(x)-f(2)}{x-2}\lim\limits_{x\to 2}\dfrac{1}{x^2+2x+4}=(-8)\left(\dfrac{1}{12}\right)=\boxed{-\dfrac{2}{3}}$

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3. $\lim\limits_{x\to 0^{\pm}}f(x)=\mp\infty\Longrightarrow\boxed{x=0 \text{ is VA}}$

$\lim\limits_{x\to 1^{\pm}}f(x)=\pm\infty\Longrightarrow\boxed{x=1 \text{ is VA}}$

$\lim\limits_{x\to\pm\infty}f(x)=\lim\limits_{x\to\pm\infty}\dfrac{|x|(x+1)}{x(x-1)}=\pm 1\Longrightarrow\boxed{y=-1 \ \& \ y=1 \text{ are HA}}$

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4. The discontinuities of f are at $\boxed{-1,\ 0,\ 2}$

$\lim\limits_{x\to -1^{\pm}}f(x)=\mp\infty\implies f$ has ID at $x=-1$

$\lim\limits_{x\to 0^{\pm}}f(x)=\pm 1\implies f$ has JD at $x=0$

$\lim\limits_{x\to 2}f(x)=\dfrac{1}{3},\ f(3)\ DNE\Longrightarrow f$ has RD at $x=2$

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5. a $f'(x)=3x^2+2x-1$

f' is continuous on $[0,1]$ and $f'(0)f'(1)<0\implies$ there exits $c\in(0,1);\ f'(c)=0$

Thus, f has horizontal tangent line at c

5. b $f'(x)=\frac{4}{3}x^{1/3}-\frac{4}{3}x^{-2/3}=\frac{4}{3}x^{-2/3}(x-1)$

f is continuous at $x=0$ and $\lim\limits_{x\to 0}|f'(x)|=\infty$

Thus, the graph of f has VTL at the point $(0,-2)$

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6. $y'=3\left[\tan\left(\dfrac{x}{2}\right)+\cos 2x\right]^2\left[\dfrac{1}{2}\sec^2\left(\dfrac{x}{2}\right)-2\sin 2x\right]$
